Lossless Image Compression Via the Lifting Scheme

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CAP 5015

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Outline

- Introduction
- Basic Idea
- Theory and Concepts
  - Split
  - Predict
  - Update
- Experiments and Results
- Summary

Notation

a_0 — Original signal

$a_{j,k}$ — Scale coefficient at level j and position k

$d_{j,k}$ — Wavelet coefficient at level j and position k

$a_{j-1,k}$ — Scale coefficient at next level: j - 1

$d_{j-1,k}$ — Wavelet coefficient at next level: j - 1

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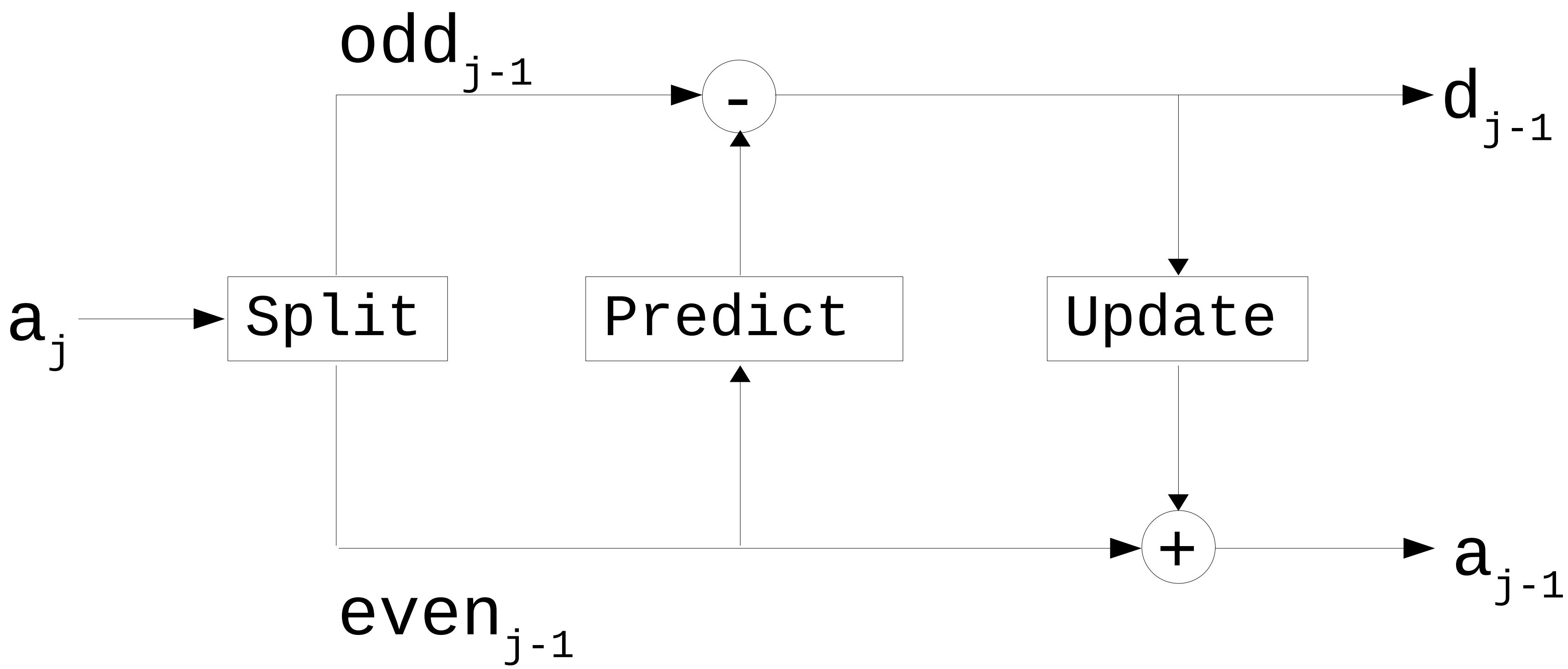
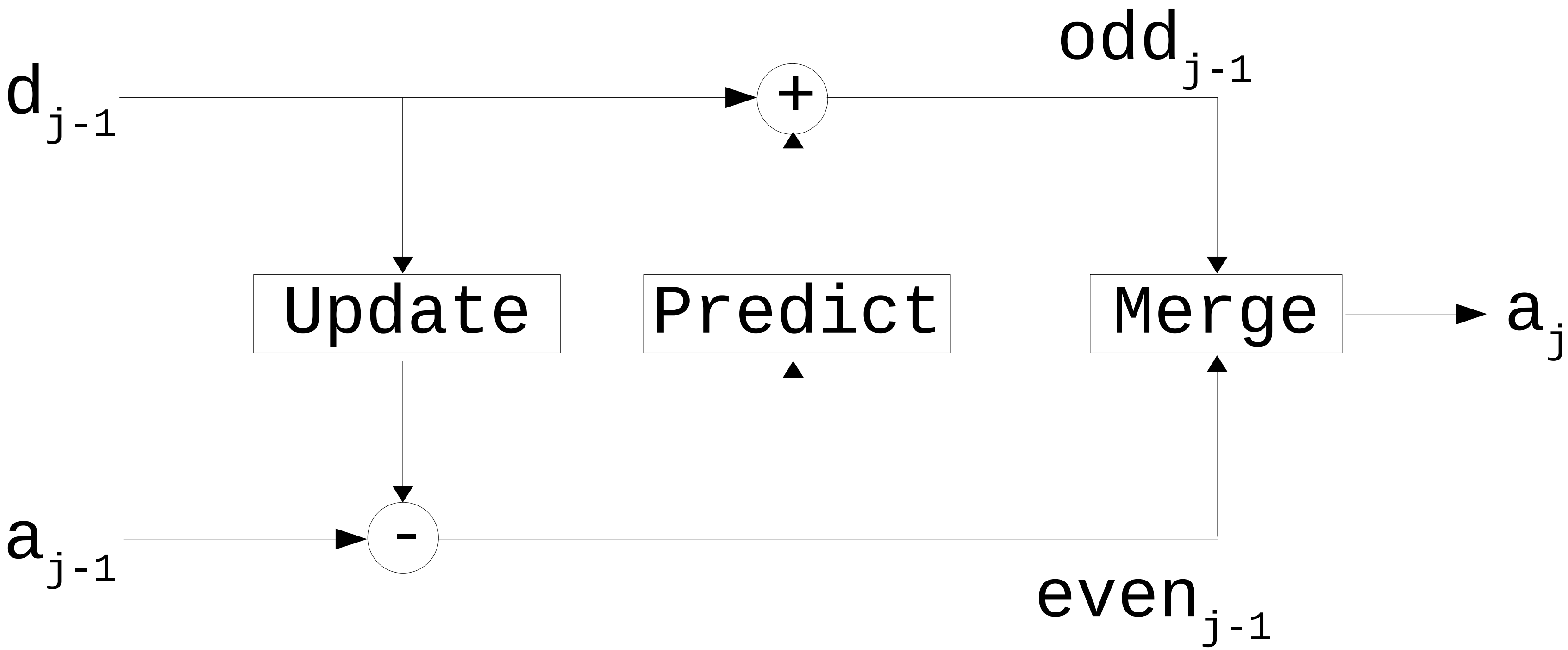
Basic Idea

Lifting Stages: Forward



Lifting Stages: Inverse

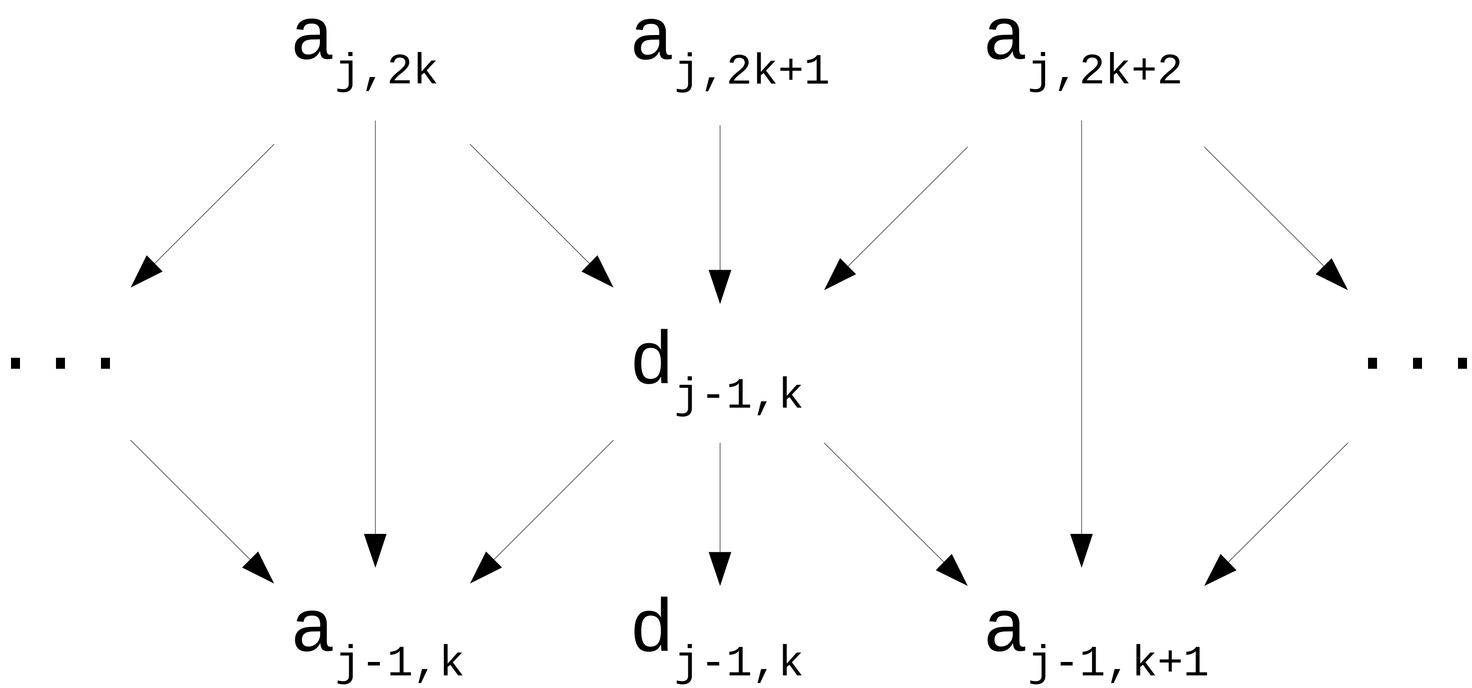


Lifting Stages: Haar

1. Split a_0 into $Even_{-1}$ and Odd_{-1}
2. $d_{-1} = Odd_{-1} - Predict(Even_{-1})$
3. $a_{-1} = Even_{-1} + Update(d_{-1})$

- $Predict(x_1) = 1/2 * (x_0 + x_2)$
- $Update(y_1) = 1/4 * (y_0 + y_2)$

In- place Transformation



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- Introduction
- Basic Idea
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  - Predict
    - Linear Prediction
    - Cubic Prediction
    - Border Issues and Filter Coefficients
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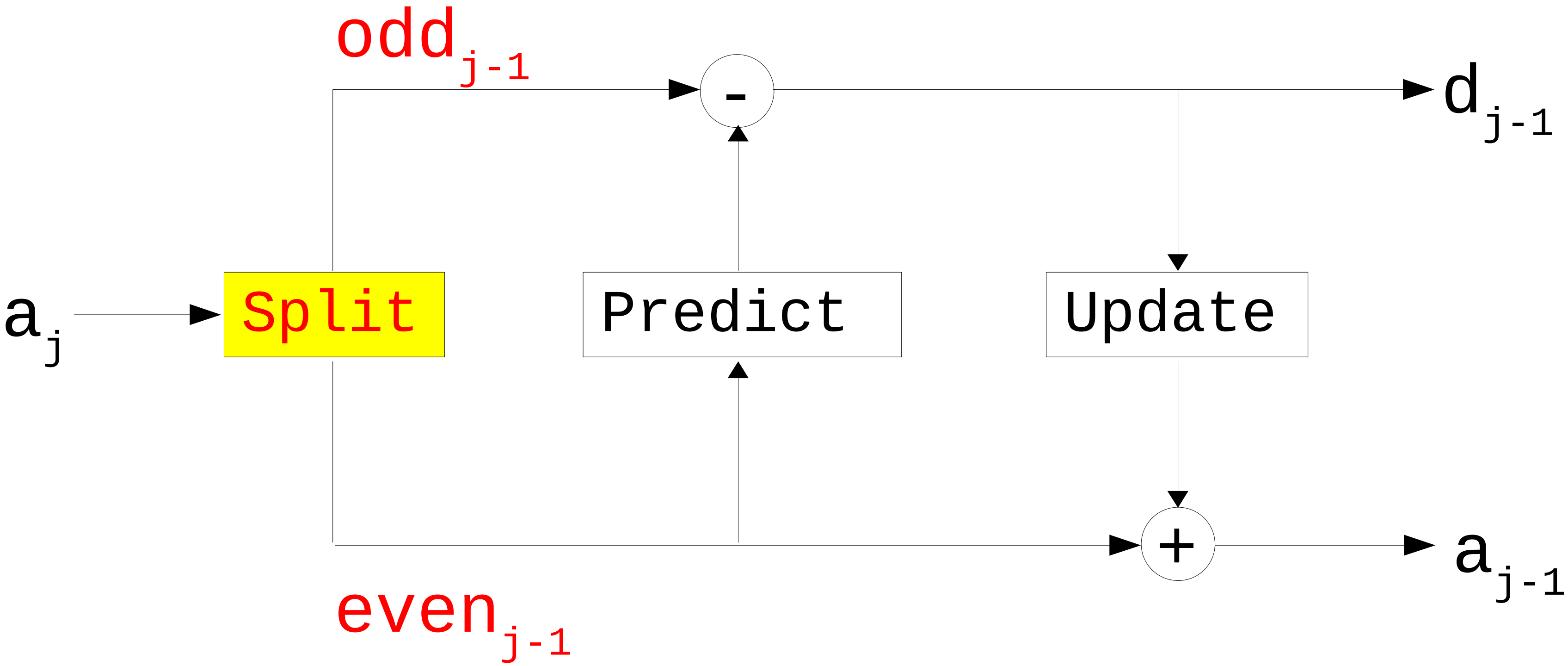
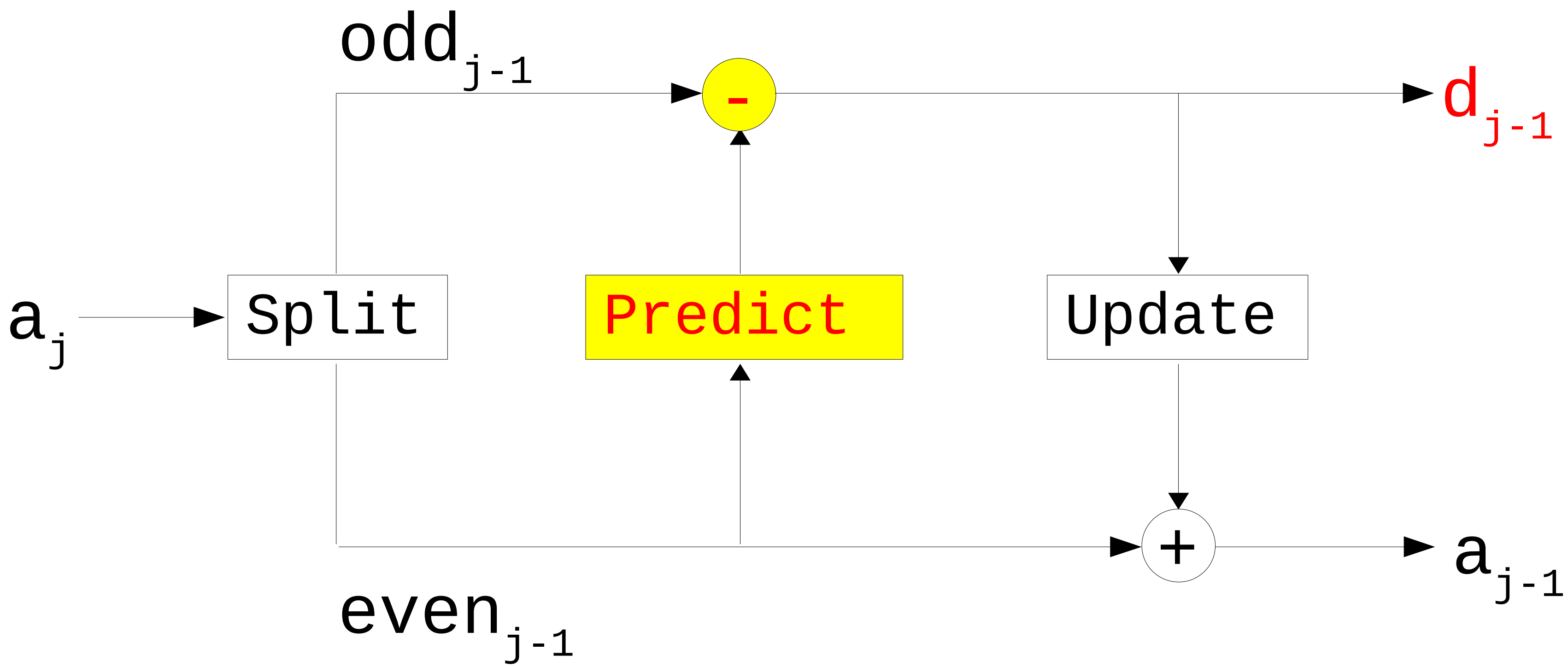
Lifting Stages: Split



$$a_{j-1,k} = a_{j,2k}$$

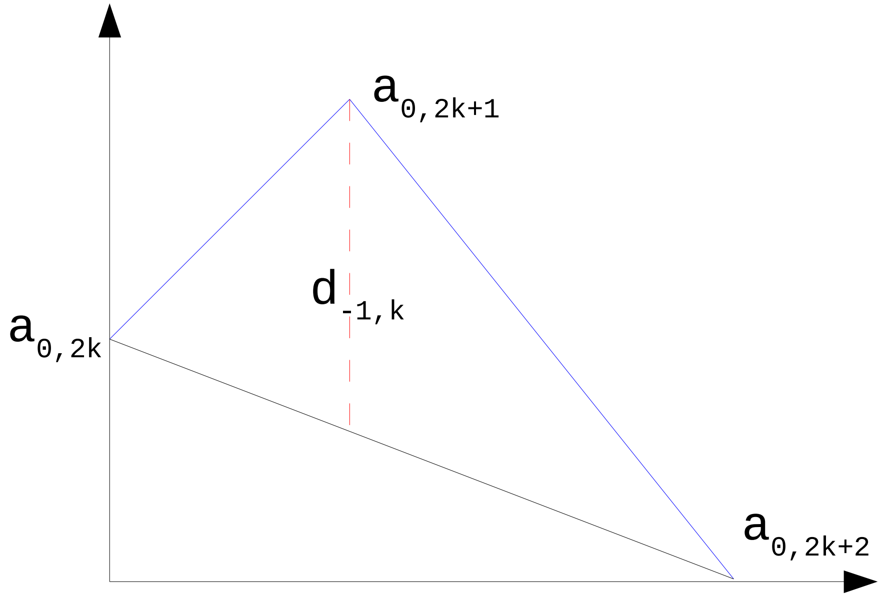
$$d_{j-1,k} = a_{j,2k+1}$$

Lifting Stages: Predict



$$d_{j-1,k} = d_{j-1,k} - \text{Predict}(a_{j-1,k})$$

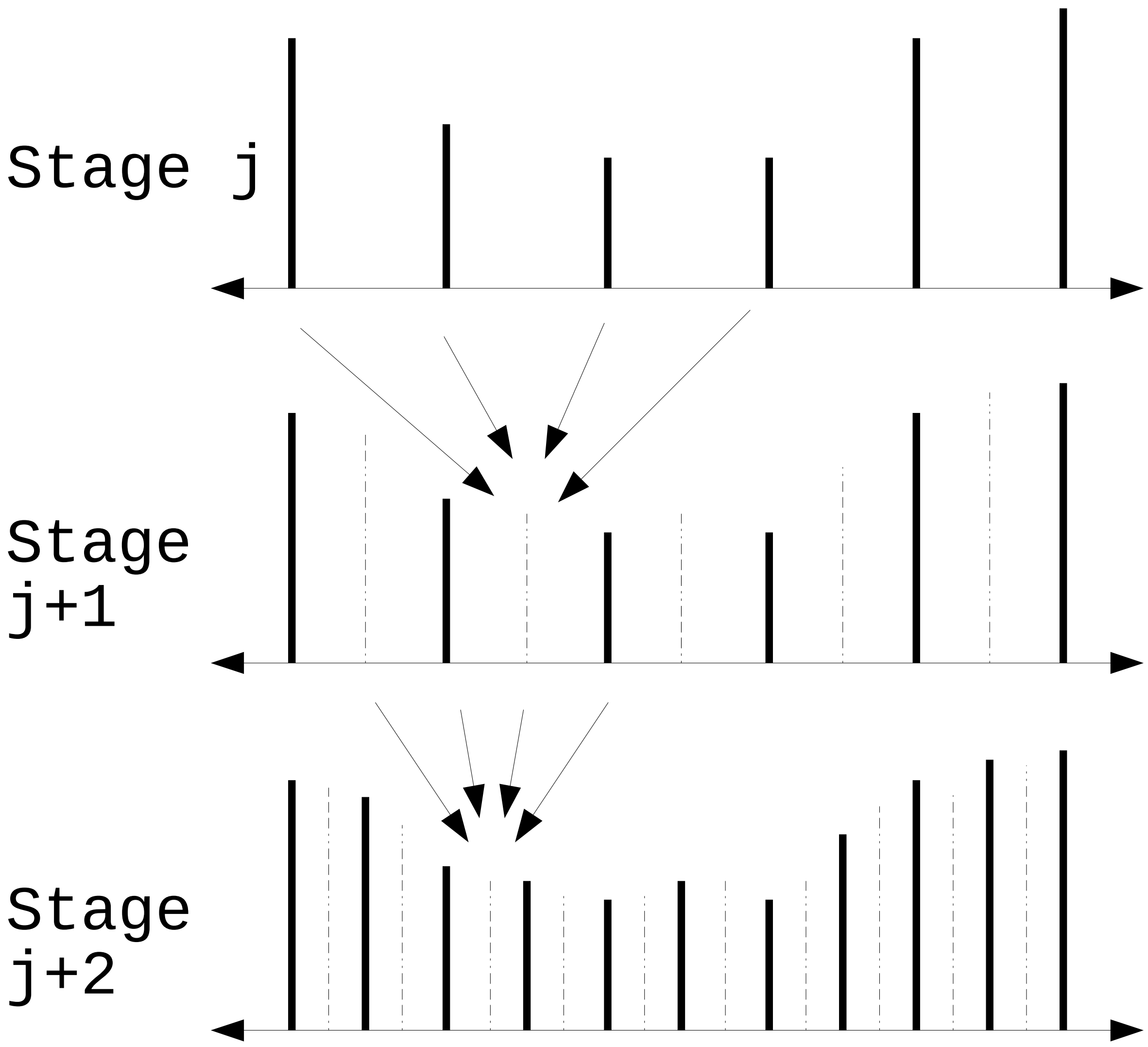
Failure to be Linear



$$d_{-1,k} = a_{0,2k+1} - Predict(a_{0,2k+1})$$

- $Predict(a_{0,2k+1}) = 1/2 * (a_{-1,k} + a_{-1,k+1})$

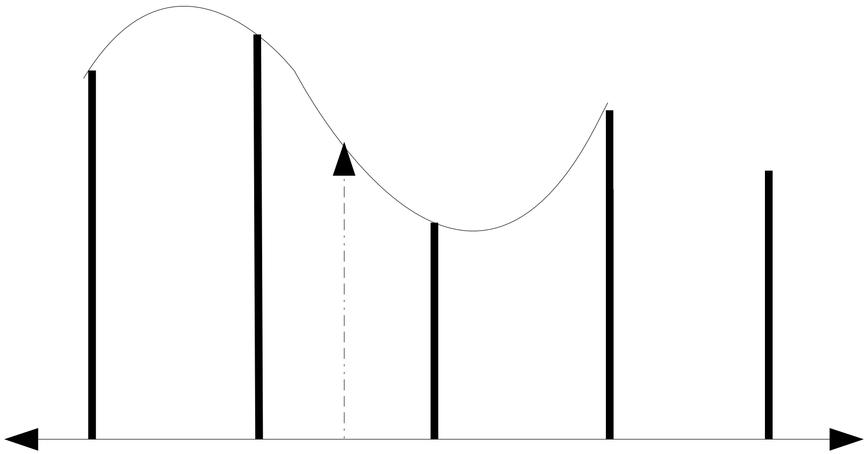
Cubic Interpolation



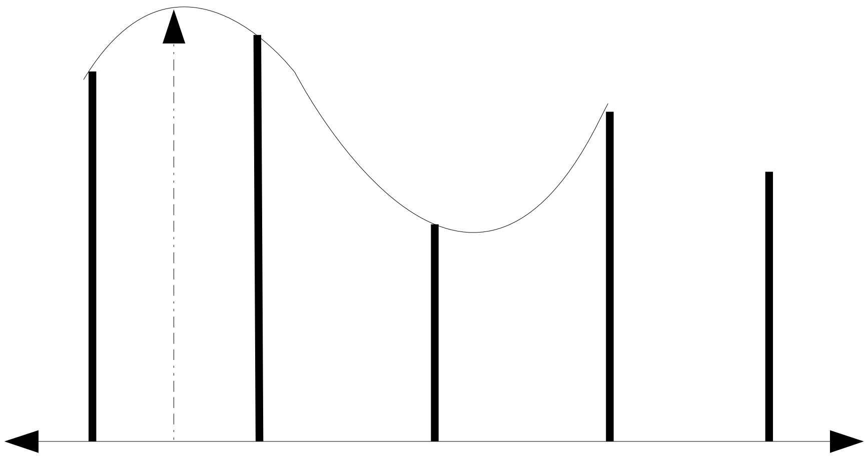
Cubic Interpolation: Neville's Algorithm

$$P_{i(i+1)\ldots(i+m)} = \frac{(x - x_{i+m}) P_{i(i+1)\ldots(i+m-1)} - (x - x_i) P_{(i+1)(i+2)\ldots(i+m)}}{x_i - x_{i+m}}$$

Border Problems

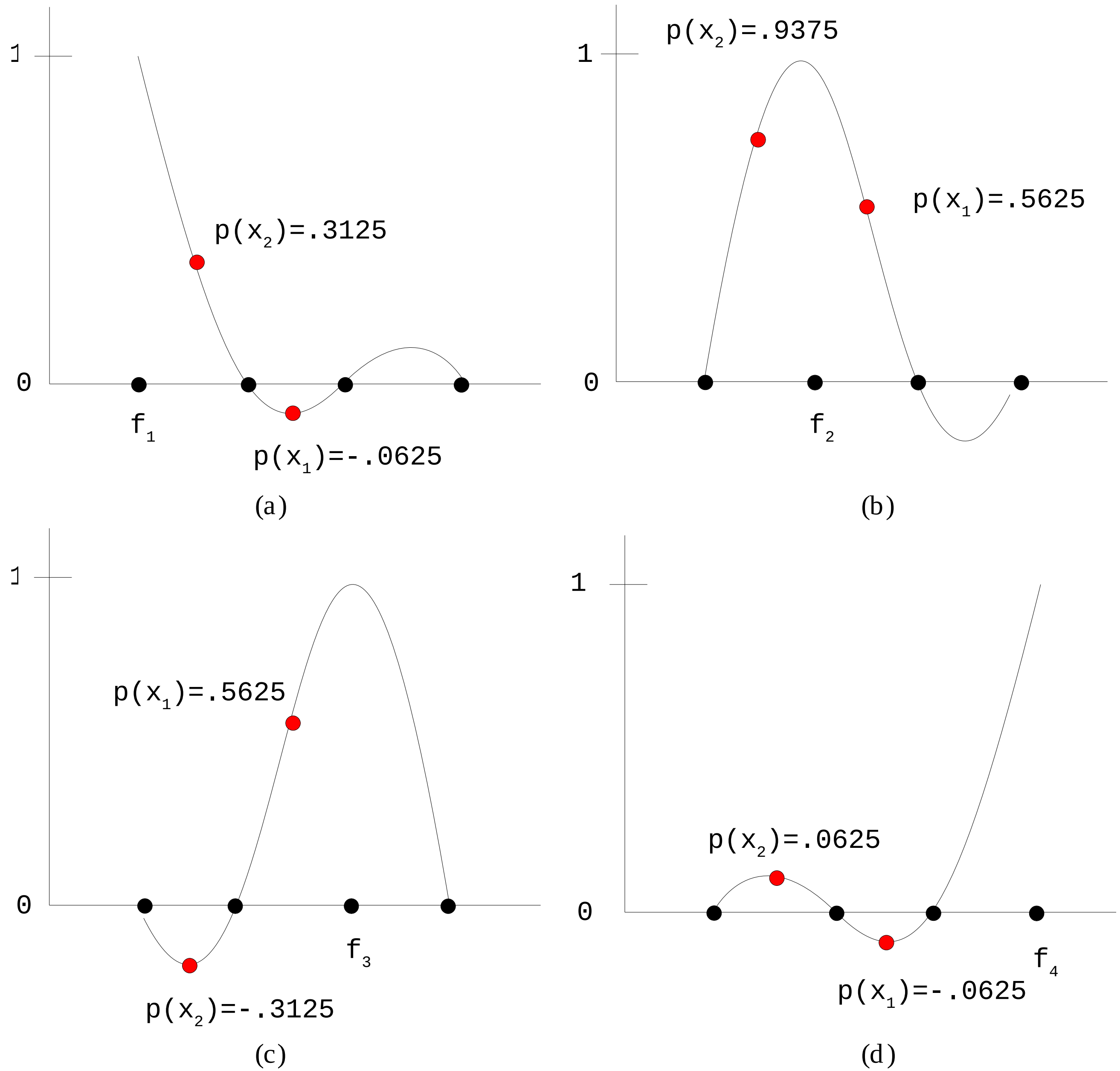


- Equal number on left and right



- Fewer on left

Border Solution



Filter Coefficients

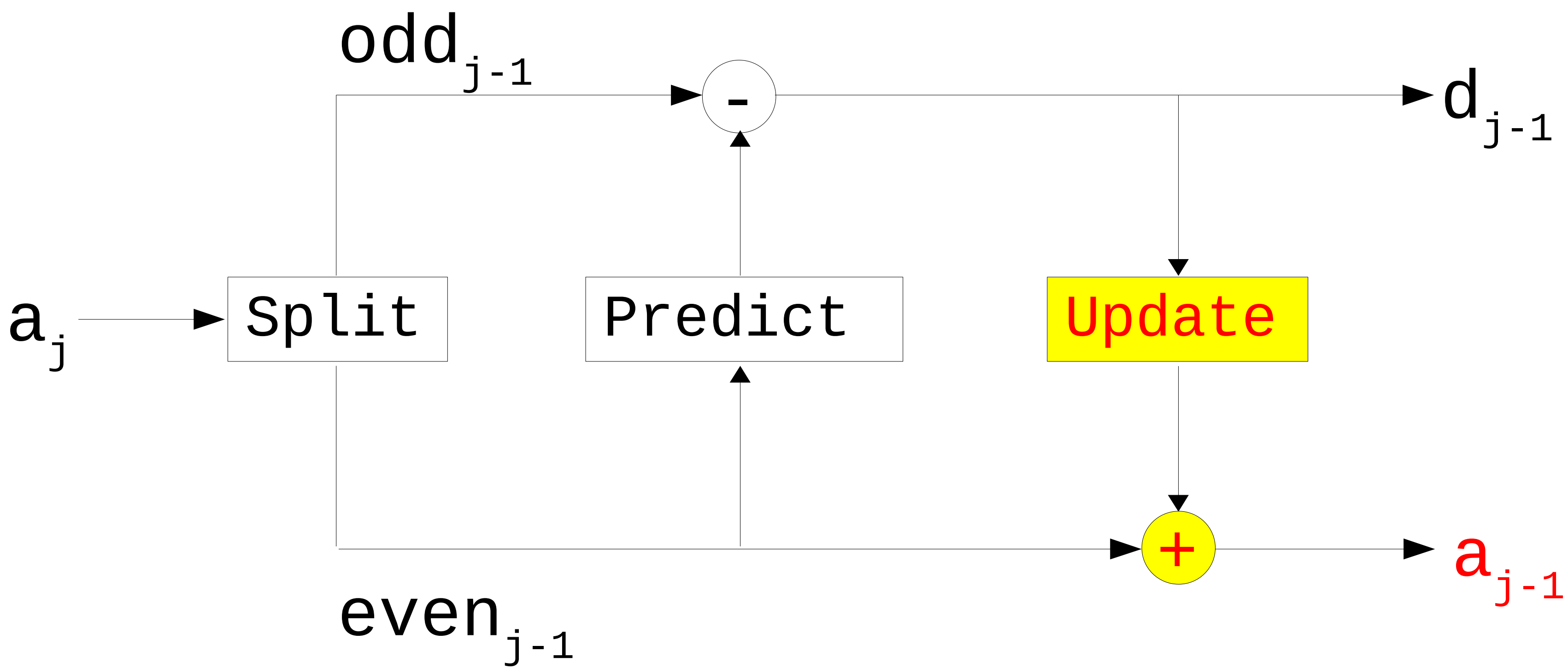
# left	# right	k - 3	k - 1	k + 1	k + 3
0	2			- .5	1.5
1	1		.5	.5	
2	0	1.5	- .5		

N= 2

# left	# right	k - 7	k - 5	k - 3	k - 1	k + 1	k + 3	k + 5	k + 7
0	4					2.1875	- 2.1875	1.3125	- .3125
1	3				.3125	.9375	- .3125	.0625	
2	2			- .0625	.5625	.5625	- .0625		
3	1		.0625	- .3125	.9375	.3125			
4	0	- .3125	1.3125	- 2.1875	2.1875				

N= 4

Lifting Stages: Update



$$a_{j-1,k} = a_{j-1,k} + \mathrm{Update}(d_{j-1,k})$$

Preserving Moments

$$\int \psi(x)\,dx = 0$$

$$\int x\psi(x)\,dx = 0$$

$$\int x^2\psi(x)\,dx = 0$$

$$\cdots$$

$$\int x^{\tilde{N}-1}\psi(x)\,dx = 0$$

Lifting Coefficients 1

- Initialize moments at each position k

Moment 1	0	1	2	3	4	5	...
Moment 2	0	1	4	9	16	25	...
Moment 3	0	1	16	27	64	125	...
...	...	...	...	...	...	...	...
Moment Ñ	0	1	$2^{Ñ}$	$3^{Ñ}$	$4^{Ñ}$	$5^{Ñ}$	...

Lifting Coefficients 2

- For each wavelet coefficient d_l
  - For each scale coefficient a_j that contributes to d_l
    - M[k, j] = M[k, j] + F[j] * M[k, l] for all k

Note:
- j is index to a coefficient $(0 < j <= N)$
- k is position for a given moment $(0 < k <= \tilde{N})$
- l is index to d coefficient (0 < l < length of signal)

Lifting Coefficients 3

Solve the system for c:

$$\begin{bmatrix} m_{l_0,0} & m_{l_1,0} & \cdots & m_{l_N,0} \\ \vdots & \vdots & \cdots & \vdots \\ m_{l_0,\tilde{N}} & m_{l_1,\tilde{N}} & \cdots & m_{l_N,\tilde{N}} \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_{\tilde{N}} \end{bmatrix} = \begin{bmatrix} m_{g_j,0} \\ \vdots \\ m_{g_j,\tilde{N}} \end{bmatrix}$$

Note:

- System is $\tilde{N} \times \tilde{N}$
- l_i is index to a coefficient
- g_j is index to d coefficient

Properties from Construction

High-Pass

- Prediction provides polynomial cancelation

Low-Pass

- Update guarantees preservation of the moments

Example

- Suppose signal a = [a_0 a_1 a_2 a_3 a_4 a_5 a_6 a_7]
- Let N = 2 and Ñ = 2

# left	***# right***	***k - 3***	***k - 1***	***k + 1***	***k + 3***
0	2			- .5	1.5
1	1		.5	.5	
2	0	1.5	- .5		

Filter Coefficients

Example: Split and Predict 1

a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7
	d_0		d_1		d_2		d_3

$$d_0 = d_0 - (.5 * a_0 + .5 * a_2)$$
$$d_1 = d_1 - (.5 * a_2 + .5 * a_4)$$
$$d_2 = d_2 - (.5 * a_4 + .5 * a_6)$$
$$d_3 = d_3 - (1.5 * a_4 + (-.5) * a_6)$$

Example: Update 1

	d_0		d_1		d_2		d_3
	2/5,1/5		0,2/3		4/15,4/5		-2/15,2/5
a_0		a_2		a_4		a_6	

$$a_0 = a_0 + (2/5 * d_0)$$
$$a_2 = a_2 + (1/5 * d_0 + 0 * d_1)$$
$$a_4 = a_4 + (2/3 * d_1 + 4/15 * d_2)$$
$$a_6 = a_6 + (4/5 * d_2 + -2/15 * d_3)$$

Example: Split and Predict 2

a_0	a_2	a_4	a_6
	d_0		d_1

$$d_0 = d_0 - (.5 * a_0 + .5 * a_4)$$
$$d_1 = d_1 - (1.5 * a_0 + (-.5) * a_4)$$

Example: Update 2

	d_0		d_1
	1/2,.214286		-1/3,.47619
a_0		a_4	

$$a_0 = a_0 + (1/2 * d_0)$$
$$a_4 = a_4 + (.214286 * d_0 + -1/3 * d_1)$$

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Experiments

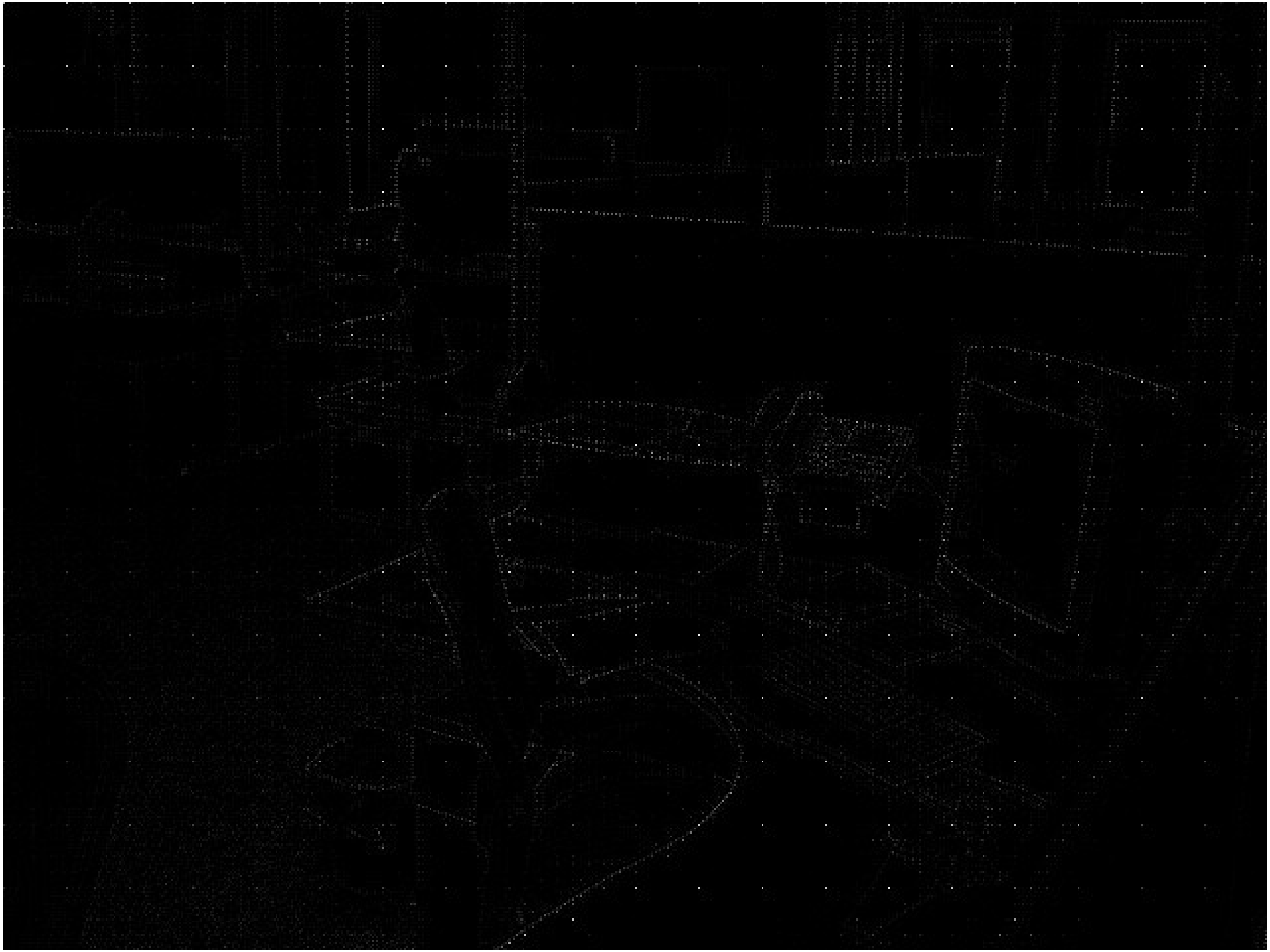
Set	*Office*	*Signs*	*Trees*
Number of images	50	50	50
Dimension	640x480	480x640	640x480
Transform Levels	5	5	5

Example images

(a) Office sample

(b) Sign sample

(c) Tree sample



Statistics Gathered

$$H(I) = -\sum p(x) * \log_2(p(x))$$

Transformation Types				
Original	Haar	(2, 2)	(4, 4)	(9, 7)

Entropy Measurements				
Average	Variance	Std. Deviation	Minimum	Maximum

Results for 'office' set

Lift Type	*Average*	*Variance*	*Std. Deviation*	*Min*	*Max*
Orignal	6.74	.34	.59	5.04	7.61
Haar	5.83	1.29	1.13	1.66	7.72
2, 2	5.16	.82	.91	1.87	6.88
4, 4	9.37	1.40	1.18	3.97	11.27
9, 7	4.42	.69	.83	1.57	5.94

Results for 'signs' set

Lift Type	*Average*	*Variance*	*Std. Deviation*	*Min*	*Max*
Orignal	6.74	.34	.59	5.04	7.61
Haar	5.83	1.29	1.13	1.66	7.72
2, 2	5.16	.82	.91	1.87	6.88
4, 4	9.37	1.40	1.18	3.97	11.27
9, 7	4.42	.69	.83	1.57	5.94

Results for 'trees' set

Lift Type	***Average***	***Variance***	***Std. Deviation***	***Min***	***Max***
Orignal	7.21	.27	.52	5.69	7.86
Haar	7.31	.53	.73	5.82	8.93
2, 2	6.68	.47	.68	5.35	8.21
4, 4	10.72	.81	.90	8.60	12.59
9, 7	5.77	.34	.58	4.59	7.11

Results for All Sets

Image Set	***Original***	***Haar***	***2, 2***	***4, 4***	***9, 7***
office	7.44	5.18	4.63	8.75	3.94
signs	6.74	5.83	5.16	9.37	4.42
trees	7.21	7.31	6.68	10.72	5.77

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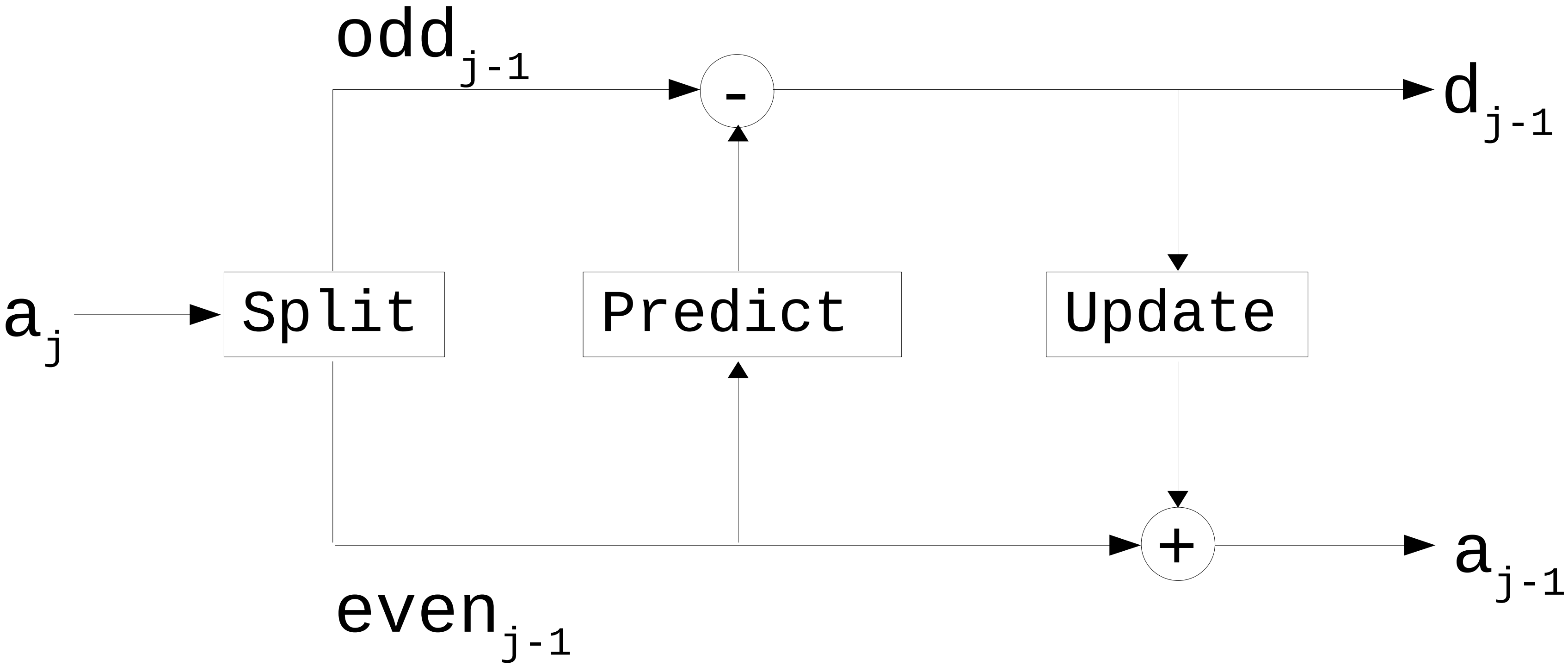

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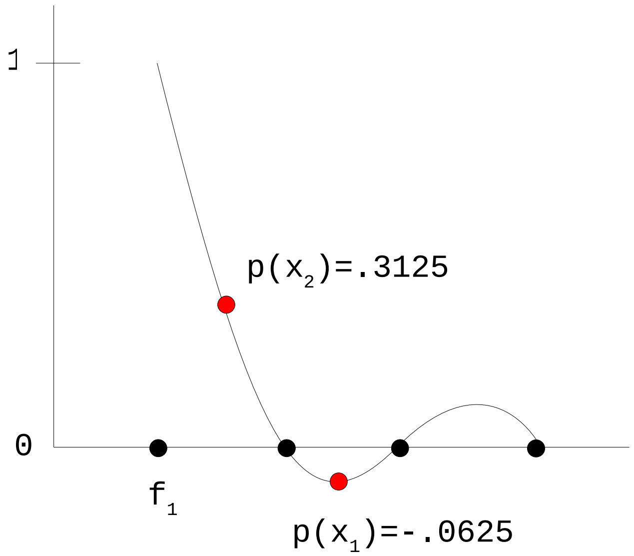

- Prediction: polynomial cancelation

$$\begin{bmatrix} m_{l_0,0} & m_{l_1,0} & \cdots & m_{l_N,0} \\ \vdots & \vdots & \cdots & \vdots \\ m_{l_0,\tilde{N}} & m_{l_1,\tilde{N}} & \cdots & m_{l_N,\tilde{N}} \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_{\tilde{N}} \end{bmatrix} = \begin{bmatrix} m_{g_j,0} \\ \vdots \\ m_{g_j,\tilde{N}} \end{bmatrix}$$

- Update: preserve moments

Summary

Image Set	***Original***	***Haar***	***2, 2***	***4, 4***	***9, 7***
office	7.44	5.18	4.63	8.75	3.94
signs	6.74	5.83	5.16	9.37	4.42
trees	7.21	7.31	6.68	10.72	5.77

Future Work

- Integer Lifting
- Object Detection

References

[1] R. Calderbank, I. Daubechies, W. Sweldens, and B.-L. Yeo. Losless image compression using integer to integer wavelet transforms. In *International Conference on Image Processing (ICIP), Vol. I,* pages5 96-599. IEEE Press, 1997.

[2] R. Calderbank, I. Daubechies, W. Sweldens, and B.-L. Yeo. Wavelet transforms that map integers to integers. *Appl. Comput. Harmon. Anal.*, 5(3):332-369, 1998.

[3] I. Daubechies and W. Sweldens. Factoring wavelet transforms into lifting steps. *J. Fourier Anal. Appl.*, 4(3):245-267, 1998.

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Questions?